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Phase Diagram of Gross-Neveu Model at Finite Temperature, Density and Constant Curvature

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Abstract

We discuss a phase structure of chiral symmetry breaking in the Gross-Neveu model at finite temperature, density and constant curvature. The effective potential is evaluated in the leading order of the $1/N$ -expansion and in a weak curvature approximation. The third order critical line is found on the critical surface in the parameter space of temperature, chemical potential and constant curvature.

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The application of finite temperature field theories is important for the restoration of electroweak symmetry and composite Higgs models, the deconfinement phase of the quark-gluon plasma and hadronic physics. The underlying concept of these approaches is a dynamical symmetry breaking and the paradigm of this notion is digested in the chiral Gross-Neveu models [1], which show the properties of renormalizability, asymptotic freedom and dimensional transmutation. The Finite temperature analyses of the Gross-Neveu models were made two decades ago [2]. The mass spectrum is given [4] and the kink-antikink configurations are not negligible in first order phase transitions [5, 6, 7]. In order to give an analytical insight into the finite temperature models and to compare with the results already obtained in various cases [2, 3], we however concentrate only on single particle state's configuration in the large N limit approach.

The chiral phase transitions caused by an external curvature are discussed in some two-dimensional models [8, 9, 10]. In this paper, we would like to examine the effective potential of the two-dimensional discrete chiral Gross-Neveu model under both finite temperature and density on the constant curvature spacetime and show its phase diagram in these external parameters' space. We first reformulate the effective potential for the Lagrangian

$$\mathcal{L} = \bar{\psi} i \gamma^\mu \nabla_\mu \psi - \frac{N}{2\lambda} \sigma^2 - \sigma \bar{\psi} \psi, \quad (1)$$

where ψ is a N -component (or -flavor) fermion field and the summation over the flavors is implicit in the fermion bilinear forms. In broken phases, the dynamical mass is generated by a vacuum expectation value of the auxiliary field σ and the phase structure is governed by the effective potential [9]

$$V(\sigma; R) = \frac{1}{2\lambda} \sigma^2 - i \text{tr} \int_0^\sigma d\sigma S(x, x) \quad (2)$$

in which $S(x, y)$ is estimated in the proper time method and

$$S(x, x) = -i \int_0^\infty ds e^{-s(\sigma^2 + R/4)} \frac{\sigma}{4\pi s} \frac{Rs/4}{sh(Rs/4)}. \quad (3)$$

With the trace formula in the Euclidean spacetime

$$1 = (4\pi s)^{D/2} \text{Tr} \exp(-s\partial^2), \quad (4)$$

we can rewrite (3) as an integral on the momentum trace so as to estimate the contribution of momenta separately from curvature's effect

$$S(x, x) = -i \int_{-\infty}^{\infty} \frac{d^2 k}{(2\pi)^2} \int_0^{\infty} ds \sigma \frac{Rs/4}{\text{sh}(Rs/4)} e^{-s(k^2 + \sigma^2 + R/4)}. \quad (5)$$

We then obtain a momentum-integral representation for the effective potential (2)

$$V(\sigma; R) = \frac{1}{2\lambda} \sigma^2 + \int \frac{d^2 k}{(2\pi)^2} \int_0^{\infty} \frac{ds}{s} \frac{Rs/4}{\text{sh}(Rs/4)} e^{-s(k^2 + R/4)} (e^{-s\sigma^2} - 1). \quad (6)$$

If we take into account of a mass term, just replace $k^2 \rightarrow k^2 + m^2$. As for an external gauge field, we may generalize (6) following the method of ref.[11] as well. This expression is also convenient to introduce the temperature as exhibited below. We here notice that the limit $R \rightarrow 0$ is now easy to obtain the usual momentum-integral representation

$$V(\sigma) = \frac{1}{2\lambda} \sigma^2 + \int \frac{d^2 k}{(2\pi)^2} \ln\left(\frac{k^2 + \sigma^2}{k^2}\right). \quad (7)$$

Before proceeding to a finite temperature model, we make some remarks here. First, in order to incorporate the temperature, we must break the general covariance and the curvature can not be defined in a covariant manner accordingly. One possible way to incorporate curvature seems to deal with the $(D-1)$ -dimensional space curvature, however such introduction of curvature in finite temperature systems becomes trivial in two-dimensional models. Although the definition of curvature would be temperature variant in the above sense, however we assume that the value of the R would be still a constant independently of the temperature. In this consideration, we can employ the constant R as an external parameter, which is already defined in original covariant theories. Second, as can be understood from the derivation of (6), the introduction of temperature on the k^2 term in (6) corresponds to a kind of flat spacetime approximation, which is slightly different from linear curvature expansions [16]. We should note also that (6) can not pick up any contribution from winding modes. There should be more controversies about

this point as well as kink-antikink mode, however, we would like to concentrate on (6) in order to separately observe its characteristic phase structure how or whether the third order critical (tri-critical) point on the T - μ plane [14] extends to the R - T - μ phase space.

We now introduce the temperature T ($= 1/\beta$) and the chemical potential μ to (6) in the usual way,

$$k^2 \rightarrow (\omega_n - i\mu)^2 + k_1^2, \quad \int \frac{dk_0}{2\pi} \rightarrow \frac{1}{\beta} \sum_n \quad (8)$$

where $\omega_n = (2n + 1)\pi/\beta$. Both integration over k_1 and summation over n can be easily performed and thus the bare potential for our finite R - T - μ model becomes

$$V(\sigma; R, \beta, \mu) = \frac{1}{2\lambda} \sigma^2 + \frac{R}{4\beta} \int_0^\infty ds \frac{\Theta_2(s \frac{2\mu}{\beta}, i s \frac{4\pi}{\beta^2})}{(4\pi s)^{1/2} \text{sh}(Rs/4)} e^{-s(R/4 - \mu^2)} (e^{-s\sigma^2} - 1), \quad (9)$$

where Θ_2 is the elliptic theta function. We can verify various limits of this proper time representation at this stage. The limit of both $\beta \rightarrow \infty$ and $\mu \rightarrow 0$ coincides with (6). For the finite T - μ effective potential we can obtain the following expression taking $R \rightarrow 0$

$$V(\sigma; \beta, \mu) = \frac{1}{2\lambda} \sigma^2 + \frac{1}{\beta} \int_0^\infty \frac{ds}{s} \frac{\Theta_2(s \frac{2\mu}{\beta}, i s \frac{4\pi}{\beta^2})}{(4\pi s)^{1/2}} e^{s\mu^2} (e^{-s\sigma^2} - 1). \quad (10)$$

This is an alternative representation to ones already obtained in [12, 14]. The limit $\beta \rightarrow \infty$ with a finite value of μ turns out

$$V(\sigma; R, \mu) = \frac{1}{2\lambda} \sigma^2 + \frac{R}{4} \int_0^\infty \frac{ds}{4\pi s} \frac{(1 + 2s\mu^2)^{-1/2}}{\text{sh}(Rs/4)} e^{-s(R/4 - \mu^2)} (e^{-s\sigma^2} - 1). \quad (11)$$

Although we should apply the renormalization condition

$$\lim_{T, \mu \rightarrow 0, R \rightarrow R_0} \frac{\partial^2}{\partial \sigma^2} V(\sigma; R, T, \mu) \Big|_{\sigma=1} = \frac{1}{\lambda_R}, \quad (12)$$

or equivalently,

$$\frac{1}{\lambda} - \frac{1}{\lambda_R} = \frac{R}{8\pi} \int_0^\infty ds \frac{1 - 2s}{\text{sh}(Rs/4)} e^{-s(1+R/4)} \Big|_{R \rightarrow R_0}, \quad (13)$$

we however put $R_0 = R$ in the following analysis. This gives same counter term up to a finite renormalization as the result of Buchbinder- Kirillova [9], and consequently phase structure does not essentially change. The renormalized effective potential of our model is therefore

$$V(\sigma; R, \beta, \mu) = \frac{1}{2\lambda_R} \sigma^2 + \frac{R}{16\beta} \int_0^\infty ds \frac{e^{-sR/4}}{\text{sh}(Rs/4)}$$

$$\times \left[\frac{1}{s} (e^{-s\sigma^2} - 1) \frac{\sqrt{4\pi s}}{\beta} e^{s\mu^2} \Theta_2\left(s\frac{2\mu}{\beta}, is\frac{4\pi}{\beta^2}\right) + \sigma^2 e^{-s}(1 - 2s) \right]. \quad (14)$$

Now we can write down the equations for the dynamical mass and the critical surface for second order transition. The gap equation is given by

$$0 = \frac{1}{\lambda_R} + \frac{R}{8\pi} \int_0^\infty ds \frac{e^{-sR/4}}{\text{sh}(Rs/4)} \left[-e^{-s(\sigma^2 - \mu^2)} \frac{\sqrt{4\pi s}}{\beta} \Theta_2\left(s\frac{2\mu}{\beta}, is\frac{4\pi}{\beta^2}\right) + e^{-s}(1 - 2s) \right]. \quad (15)$$

Hereafter we adopt the value $\lambda_R = \pi$ for the renormalized coupling constant of which value coincides with that of $T = \mu = R = 0$ limit case. The critical surface on which the second order phase transition occurs is described by

$$\lim_{\sigma \rightarrow 0} \frac{\partial}{\partial \sigma^2} V(\sigma; R, \beta, \mu) = 0, \quad (16)$$

namely, by the following equation

$$0 = \int_0^\infty \frac{ds}{4\pi s} \left[e^{-sR/4} \frac{Rs/4}{\text{sh}(Rs/4)} \left\{ -e^{s\mu^2} \frac{\sqrt{4\pi s}}{\beta} \Theta_2\left(s\frac{2\mu}{\beta}, is\frac{4\pi}{\beta^2}\right) + e^{-s}(1 - 2s) \right\} + 2se^{-s} \right]. \quad (17)$$

We should note that this surface covers up the first order critical surface in some region and becomes irrelevant to describe the second order phase transition (for instance see Fig.1). This situation is very similar to that of T - μ critical line [14].

We can see that some known results are recovered again in particular limits. First, let us consider the case of limit $R \rightarrow 0$. Each term in the above integrand diverges in this case. Introducing a parameter D to regularize these integrals as

$$0 = \int_0^\infty \frac{ds}{(4\pi s)^{D/2}} \left[e^{-s} - e^{s\mu^2} \frac{\sqrt{4\pi s}}{\beta} \Theta_2\left(s\frac{2\mu}{\beta}, is\frac{4\pi}{\beta^2}\right) \right], \quad (18)$$

we obtain

$$\beta^{D-2} \Gamma\left(1 - \frac{D}{2}\right) = \frac{2}{\sqrt{\pi}} (2\pi)^{D-2} \Gamma\left(\frac{3-D}{2}\right) \Re \zeta\left(3-D, \frac{1}{2} + i\frac{\beta\mu}{2\pi}\right), \quad (19)$$

where ζ is the generalized zeta function. This is nothing but the equation for the second order critical line on the T - μ plane in the dimensional regularization formalism [12], in which this equation is proved to reproduce Trembl's result [13] in the limit $D \rightarrow 2$. Second, let us take the limit both $\beta \rightarrow \infty$ and $\mu \rightarrow 0$ in Eq.(17);

$$0 = \int_0^\infty \frac{ds}{4\pi s} \left[e^{-sR/4} \frac{Rs/4}{\text{sh}(Rs/4)} \{-1 + e^{-s}(1 - 2s)\} + 2se^{-s} \right]. \quad (20)$$

This becomes

$$2 = \gamma + \psi(1 + \frac{2}{R}) + \frac{4}{R}\psi'(1 + \frac{2}{R}), \quad (21)$$

where γ is the Euler constant and ψ the digamma function. Buchbinder and Kirillova's result is now recovered using asymptotic expansion for $R \sim 0$ [9];

$$\gamma = \ln(\frac{R}{2}). \quad (22)$$

The limit $\beta \rightarrow \infty$ becomes the second order critical line on R - μ plane (Fig.2);

$$0 = \int_0^\infty \frac{ds}{4\pi s} [e^{-sR/4} \frac{Rs/4}{\text{sh}(Rs/4)} \{ -\frac{e^{s\mu}}{\sqrt{1+2s\mu^2}} + e^{-s}(1-2s) \} + 2se^{-s}], \quad (23)$$

which can be cast into the following simple form similarly to (21)

$$0 = 2 + \psi_d(\frac{2}{R}; \frac{2}{R}\mu^2) + \frac{4}{R}\psi'_d(\frac{2}{R}; \frac{2}{R}\mu^2), \quad (24)$$

where ψ'_d means a derivative on z of the function

$$\psi_d(z; a) \equiv \int_0^\infty ds \frac{e^{-sz} - e^{sa}(1+2as)^{-1/2}}{e^s - 1}. \quad (25)$$

As in the literature [14] (see also [12]), we observe the tri-critical point on the T - μ plane. Our interest is to examine how the tri-critical point extends to finite curvature region as previously mentioned. In our computation, the determination of the third order critical line owes to the following equation;

$$\lim_{\sigma \rightarrow 0} (\frac{\partial}{\partial \sigma^2})^2 V(\sigma; R, \beta, \mu) = 0. \quad (26)$$

The explicit expression (integral representation) for (26) is

$$0 = \int_0^\infty ds \frac{se^{s(\mu^2-R/4)}}{\text{sh}(Rs/4)} \frac{\sqrt{4\pi s}}{\beta} \Theta_2(s\frac{2\mu}{\beta}, is\frac{4\pi}{\beta^2}). \quad (27)$$

The intersection between this surface (27) and the surface (17) is just the tri-critical line which cuts away an irrelevant piece from the surface (17) and defines the division between the first and the second order phase transitions. Fig.1 shows the determination of a tri-critical point at $\mu = 0.65$. The tri-critical point **C** is found from crossing between two lines **A** and **B** which correspond to (17) and (27) respectively.

The surface (27) reduces the following equation of line on the R - μ plane

$$0 = \int_0^\infty ds \frac{se^{sz}}{e^s - 1} (1 + 2sz)^{-1/2}, \quad z = \frac{2}{R}\mu^2, \quad (28)$$

however this line does not intersect with the second order critical line (24) in the region $\mu \leq 1$ on the R - μ plane. There is thus no tri-critical point in that region. In Fig.2, we draw the second order critical line (24). It monotonically extends to the point $(\mu, R) = (1, 3.84)$, which we abbreviated in the figure.

Now we entirely explain how we draw the phase diagram of our model. First, we start from a well-known point on the T - μ plane. The tri-critical point can be found from (18) and the surface (27) with the limit $R \rightarrow 0$

$$0 = \int_0^\infty ds e^{s\mu^2} \frac{\sqrt{4\pi s}}{\beta} \Theta_2\left(s\frac{2\mu}{\beta}, is\frac{4\pi}{\beta^2}\right), \quad (29)$$

which coincides with the equation [14]

$$\Re\zeta\left(3, \frac{1}{2} + i\frac{\beta\mu}{2\pi}\right) = 0. \quad (30)$$

For finite curvature case, we similarly repeat this kind of analysis using (17) and (27) to obtain the third order critical line. Next, with the aid of (17), we can draw the second order critical surface. Finally, we only have to determine the surface where the first order phase transition occurs. Varying the parameter T for fixed R and μ , we carefully observe the shapes of the effective potential (14) until the symmetry restores.

The results are summarized in Fig.3. The Critical lines at $\mu = 0, 0.5, 0.6, 0.65, 0.7, 0.8, 1.0$ are depicted. Increasing the value of μ from zero, we firstly find one tri-critical point at $(T, R) = (0.25, 0.7)$ when $\mu = 0.585$. After then, we observe two tri-critical points at both edges of the first order critical line (for example, see $\mu = 0.6$ case in Fig.3) until $\mu = 0.608$. When $\mu = 0.608$, we obtain two tri-critical points $(0.21, 1.1)$ as well as $(0.32, 0)$ (the latter point is well-known result). For the interval after $\mu = 0.608$ to $\mu = 1$, we have only one tri-critical point for each value of μ . The tri-critical line would penetrate $R - \mu$ plane in the region $1 < \mu$.

We have observed the behaviour of the effective potential in the large N leading order as well as in a flat spacetime approximation and have showed the third order critical

line in the R - T - μ space. In spite of these approximations, we have obtained a smooth interpolation between critical values of temperatures and curvatures. For example, when $\mu = 0$, the point $T_c = 0.57$ is smoothly connected to the point $R_c = 2.6$. As a result of this feature, the tri-critical line is extended into finite R region smoothly.

In higher dimensional Gross-Neveu models, the chiral symmetry breaking is of second order ($2 \leq D < 4$) in the large N approach [12, 15] while the first order transition ($D = 3, 4$) appears in the presence of curvature [16]. Also in a lattice calculation (without curvature), the existence of first order transition is reported in three dimensions [17]. It would be interesting whether the dimensional extension of our model would give an analytical or quantitative connection between these kind of transitions in 3 or 4 dimensions. The Massive Gross-Neveu model is another interest of our approach. The addition of a slight mass term does not entirely change the phase structure of the finite T - μ model excepting that the second order transition disappears [18]. We would like to pay attention to what our model would have an effect on this point.

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FIGURE CAPTIONS

Fig.1: Determination of the tri-critical point **C** at $\mu = 0.65$. The **C** is determined from crossing of the second order critical surface **A** and the surface **B**. Dashed line **D** is the first order critical line.

Fig.2: Phase structure on R - μ plane. **A** and **D** follow the notation of Fig.1. Region **S** is the symmetric phase and **B** the broken phase.

Fig.3: Critical lines at various values for μ .

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